Introduction to Quantum Noise

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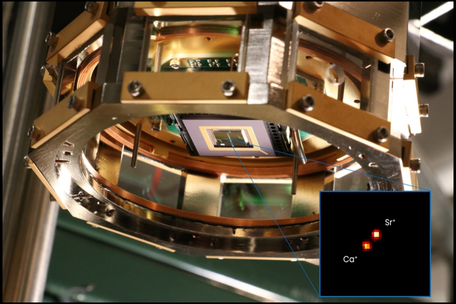
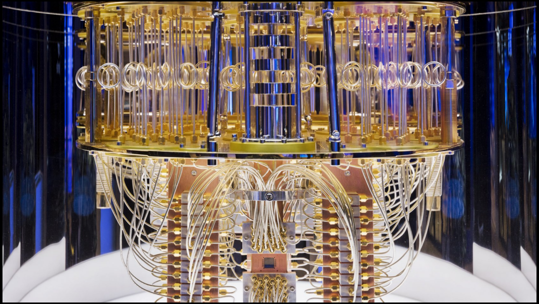
Why?





Left: Trapped-Ion (Lincoln Laboratory). Right: Superconducting (IBM Quantum)

Why?

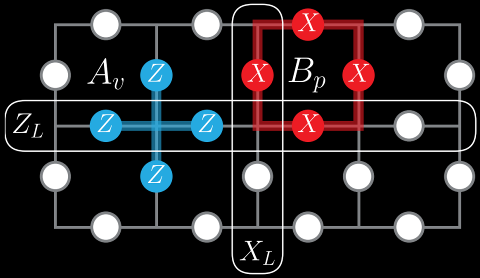
Quantum computers are susceptible to noise from various sources, like disturbances in Earth's magnetic field, local radiation from Wi-Fi or mobile phones, cosmic rays, and even the influence from neighboring qubits.

Why?

- Noise makes quantum computers inaccurate
- Even worse, noise causes quantum computers to *lose* information
- Today’s quantum computers are *noisy*
- Tomorrow’s quantum computers will be *noisy*

Quantum Error Correction

Entangle many qubits together to *spread* the information.



Example: *Surface codes* encode two logical qubits using a lattice of physical qubits

Quantum Error Correction

Quantum Threshold Theorem: *quantum error correction is viable when physical qubit error rates are below a certain threshold*

Dilemmas:

- Need *reliable* qubits
- Need lots of qubits

Quantum error correction will not be viable anytime soon 😞

What can we do?

Let’s try to *characterize* quantum noise

Let’s try to *mitigate* and *suppress* quantum noise

“The road to advantage” - ibm.com/quantum/roadmap

“Our quantum computing journey” - quantumai.google/learn/map

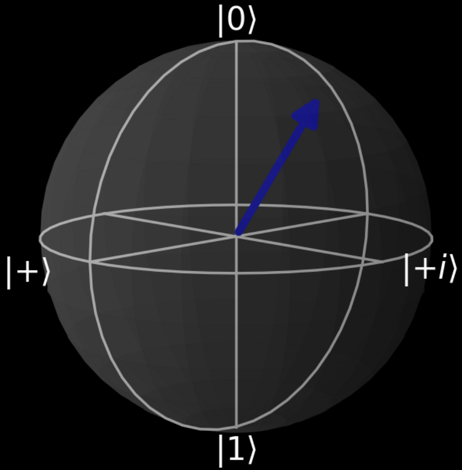
Overview

1. Notation and State Initialization Errors
2. Coherent Errors
3. Incoherent Errors
4. Decoherent Errors

Notation

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$



Notation

Quantum computers are *real* quantum systems!

$$i\frac{\partial}{\partial t}\left|\psi\right\rangle = \hat{H}(t)\left|\psi\right\rangle, \quad i\frac{\partial}{\partial t}\rho = [\hat{H}(t), \rho]$$

$$\left|\psi(t_f)\right\rangle = U(t_f, t_i)\left|\psi(t_i)\right\rangle$$

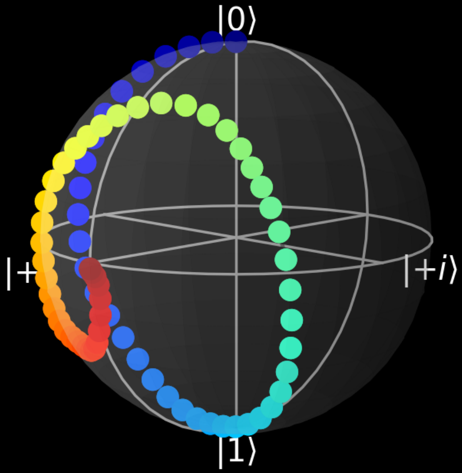
$$U(t_f, t_i) = \mathcal{T}\exp\left(-i\int_{t_i}^{t_f}\hat{H}(t)dt\right)$$

Example 1

How can we get the
$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ state?

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$H|0\rangle = |+\rangle$$



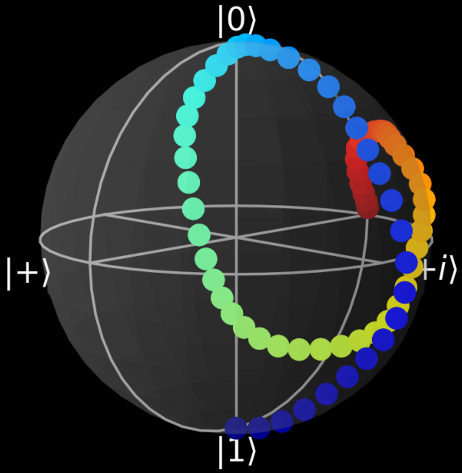
Example 1

We obtained $|+\rangle$ by assuming initial state $|0\rangle$. What happens if we started in some other state?

Example 1

What if we started at $|1\rangle$?

$$H\,|1\rangle = |-\rangle = \frac{1}{\sqrt{2}}\,(|0\rangle - |1\rangle)$$



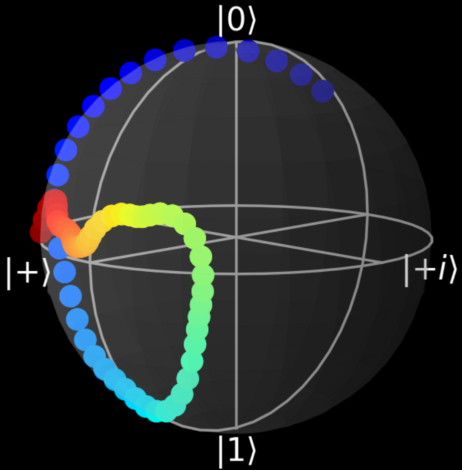
Example 1

How about

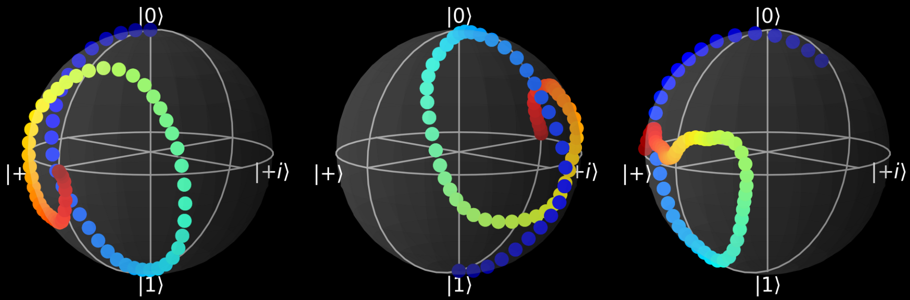
$$|\psi_r\rangle = \frac{1}{\sqrt{10}}\left(3i\,|0\rangle - |1\rangle\right)?$$

$$H\,|\psi_r\rangle =$$

$$\frac{1}{2\sqrt{5}}\left[(-1+3i)\,|0\rangle + (1+3i)\,|1\rangle\right]$$



Example 1



How can we combine these possibilities?

Example 1

Suppose we have some probabilities for each initial state:

- 90%: $|0\rangle$
- 8%: $|1\rangle$
- 2%: $|\psi_r\rangle = \frac{1}{\sqrt{10}}\left(3i\,|0\rangle - |1\rangle\right)$

It would be nice to have some mathematical expression to express the possible measurement outcomes!

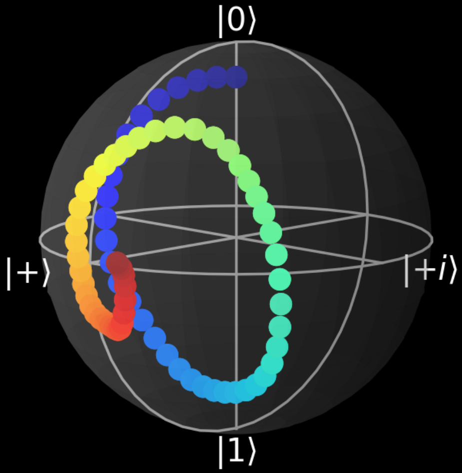
Example 1 - Density States

$$\rho = \sum_i p_i \left|\psi\right\rangle \left\langle\psi\right|$$

$$\rho = 0.9 \left|0\right\rangle \left\langle 0\right| + 0.08 \left|1\right\rangle \left\langle 1\right| + 0.02 \left|\psi_r\right\rangle \left\langle\psi_r\right|$$

Example 1 - Density States

$$\rho' = H \rho H^\dagger$$



Overview

1. ~~Notation and State Initialization Errors~~
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Example 2

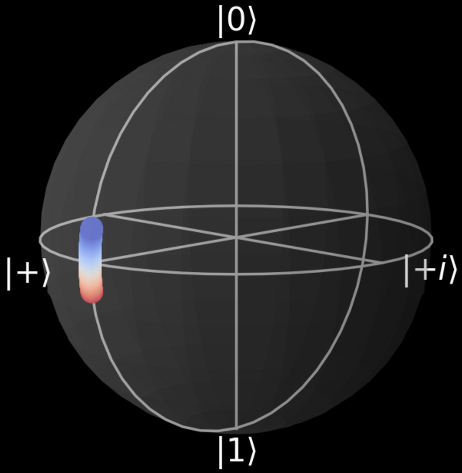
What if our H gate is slightly incorrect?

$$\tilde{H} = U(\delta) \circ H$$

Example 2

$$R_x(\delta\theta) = \exp(-i\delta X)$$

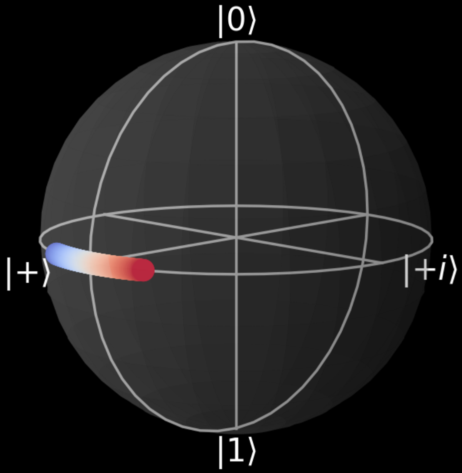
$$\tilde{H} = R_x(\delta) \cdot H$$



Example 2

$$R_z(\delta\theta) = \exp(-i\delta Z)$$

$$\tilde{H} = R_z(\delta) \cdot H$$



Coherent Errors

Coherent errors are fixed *unitary* operations, e.g.: $\tilde{H} = U(\delta) \circ H$

Coherent Errors

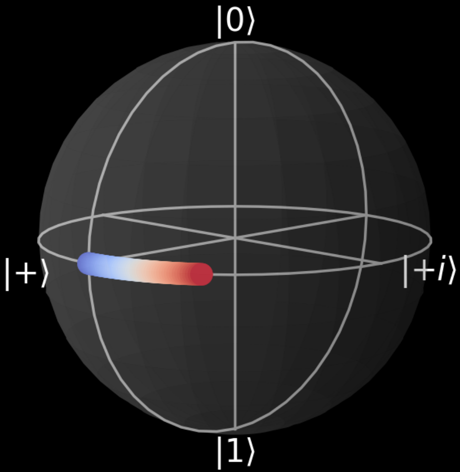
- These can be quite damaging, because they accumulate in one “direction”
- A common example are “coherent phase errors” which are characterized by T2 time

Coherent Error Mitigation: Dynamical Decoupling

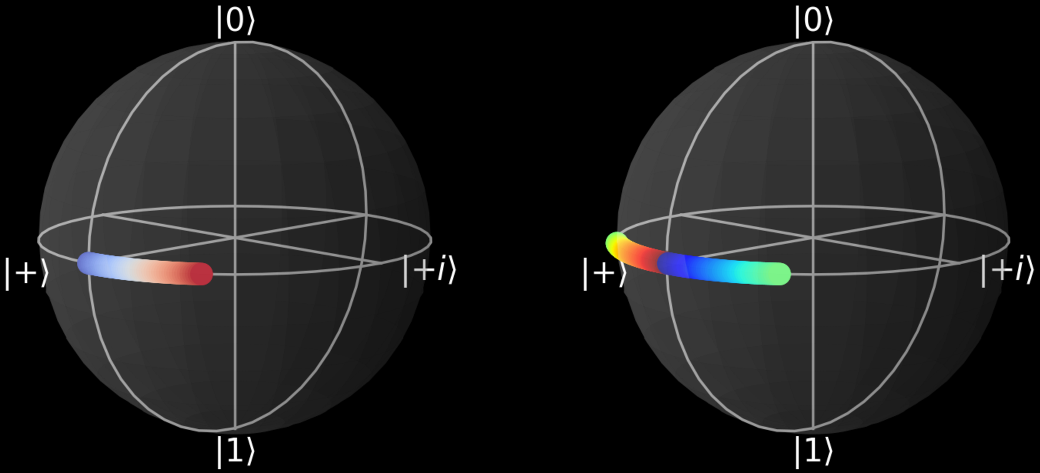
Key Idea:

- Periodically rotate the qubit state back and forth
- This causes the coherent errors to cancel out

Coherent Error Mitigation: Dynamical Decoupling



Coherent Error Mitigation: Dynamical Decoupling



Coherent Error Mitigation: Dynamical Decoupling

Limitations:

- Imperfect control pulses
- Finite pulse durations
- Need to calibrate the time intervals

Overview

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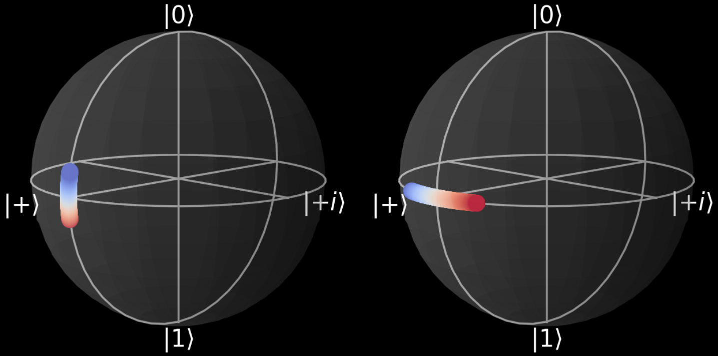
Example 3

Suppose we have two options:

- 30%: $U(\delta)$
- 50%: $U(\gamma)$



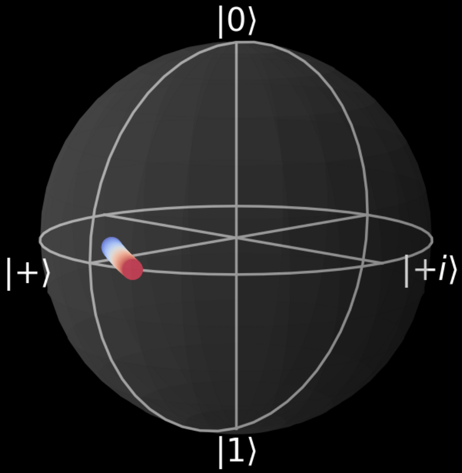
Example 3

What is the average effect?

$$\mathcal{E}(\rho) = \int p(\delta) d\delta U(\delta) \rho U^\dagger(\delta) + \int p(\gamma) d\gamma U(\gamma) \rho U^\dagger(\gamma)$$

$$\mathcal{E}(\rho) = 0.3U(\delta)\rho U^\dagger(\delta) + 0.5U(\gamma)\rho U^\dagger(\gamma)$$

Example 3



Incoherent Errors

- Probability distribution over unitary operations
- The apparent non-unitary behavior is due to the distribution being over external experiential parameters.
- Can still mitigate them using good control techniques

Overview

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Decoherent Errors

What if there are no distributions of unitary operators that describe the noise?

Example 3

The “probabilities” vary over time:

$t = 0\Delta t$

- 30%: $U(\delta)$
- 50%: $U(\gamma)$

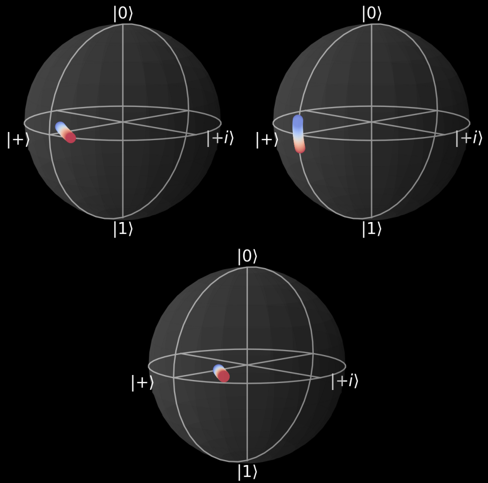
$t = 1\Delta t$

- 90%: $U(\delta)$
- 10%: $U(\gamma)$

$t = 2\Delta t$

- 20%: $U(\delta)$
- 20%: $U(\gamma)$

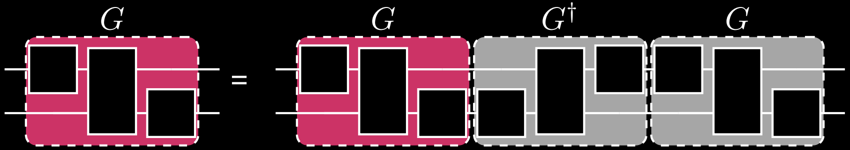
...



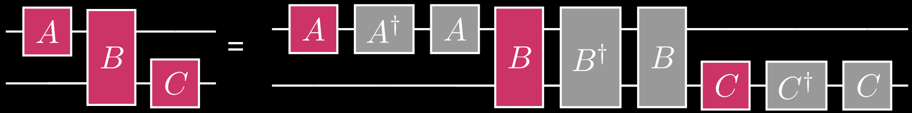
Decoherent Error Mitigation

1. Try to approximate an incoherent error
2. ~~Quantum Error Correction~~
3. Let a classical computer “learn”

Zero Noise Extrapolation: Global Strategy



Zero Noise Extrapolation: Local Strategy



Zero Noise Extrapolation: Pulse Streching

Zero-noise Extrapolation

1. Let τ quantify the noise-level in a circuit.
2. Intentionally increase circuit depth, $\tau' = \lambda\tau$
3. Fit a function $f(\lambda; a_0, a_1, ...)$
4. Extrapolate at "zero noise" $\lambda = 0$

Zero-noise Extrapolation

Pros:

- Do not need detailed knowledge of a noise model

Cons:

- May suffer from large bias – a weak-formulated function f, such as a low-degree polynomial, may not achieve the zero-noise limit.
- For circuit with short depth, the lowest error points may be dominated by noise and perform worse than the unmitigated result.

Conclusion

We looked at various kinds of quantum errors:

1. Coherent
2. Incoherent
3. Decoherent

and strategies to mitigate them.

Next time: how to characterize quantum noise?